TELANGANA RESIDENTIAL EDUCATIONAL INSTITUTIONS RECRUITMENT BOARD TREI-RB

Notations :

1.Options shown in green color and with ✔ icon are correct.

2.Options shown in red color and with ✖ icon are incorrect.

Question Paper Name :	Mathematics 6th Aug 2023 Shift 2
Subject Name :	Mathematics
Creation Date :	2023-08-06 15:25:02
Duration :	120
Total Marks :	100
Display Marks:	Yes
Calculator :	None
Magnifying Glass Required? :	No
Ruler Required? :	No
Eraser Required? :	No
Scratch Pad Required? :	No
Rough Sketch/Notepad Required? :	No
Protractor Required? :	No
Show Watermark on Console? :	Yes
Highlighter :	No
Auto Save on Console?	Yes
Change Font Color :	No
Change Background Color :	No
Change Theme :	No
Help Button :	No

Show Reports :	No
Show Progress Bar :	No

Mathematics

Group Number :	1
Group Id :	59425348
Group Maximum Duration :	0
Group Minimum Duration :	120
Show Attended Group? :	No
Edit Attended Group? :	No
Break time :	0
Group Marks :	100
Is this Group for Examiner? :	No
Examiner permission :	Cant View
Show Progress Bar? :	No

Mathematics

Section Id :	59425356
Section Number :	1
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	100
Number of Questions to be attempted :	100
Section Marks :	100
Enable Mark as Answered Mark for Review and Clear Response :	Yes
Maximum Instruction Time :	0
Sub-Section Number :	1

Sub-Section Id :	59425366
Question Shuffling Allowed :	Yes
Is Section Default? :	null

Question Number : 1 Question Id : 5942534711 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

$5^{1192} \pmod{28}$ is equal to

Options :

1. ✖ $3 \pmod{28}$

2. ✔ $9 \pmod{28}$

3. ✖ $11 \pmod{28}$

4. ✖ $27 \pmod{28}$

Question Number : 2 Question Id : 5942534712 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the statements.

S_1 : There does not exist a natural number n which gives remainder 1 when divided by 9, 25 and 49.

S_2 : If $(n, m) = 1$ then $(\phi(n), \phi(m))=1$.

Then

Options :

1. ✔ both S_1 and S_2 are false

2. ✖ S_1 is true but S_2 is false

3. ✖ S_1 is false but S_2 is true

4. ✖ both S_1 and S_2 are true

Question Number : 3 Question Id : 5942534713 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let a and b be natural numbers and suppose $7^{a-7} - 7^{b-7} = 2400$. Then

Options :

1. ✔ $a = 11,\ b = 7$

2. ✖ $b = 7$ but a cannot be determined

3. ✖ $a = 12,\ b = 8$

4. ✖ the information is not enough for a and b to be determined

Question Number : 4 Question Id : 5942534714 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum

Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let p be an odd prime and $a_1, a_2, \ldots, a_{p-1}$ a complete set of non-zero residues mod p. Then $(p-1)!a_1a_2....a_{p-1}$ is equal to

Options :

1. ✔ $1(\text{mod } p)$

2. ✖ $-1(\text{mod } p)$

3. ✖ $0(\text{mod } p)$

4. ✖ $2(\text{mod } p)$

Question Number : 5 Question Id : 5942534715 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the statements.

S_1 : If p, q are primes with $p < q$ and $n = pq$ then $q - 1$ cannot divide $n - 1$.

S_2 : If p, q are prime numbers and a, b non-negative integers then the number of divisors of $p^a q^b$ are $(a+1)(b+1)$.

Then

Options :

1. ✖ both S_1 and S_2 are false

2. ✖ S_1 is true but S_2 is false

3. ✖ S_1 is false but S_2 is true

4. ✔ both S_1 and S_2 are true

Question Number : 6 Question Id : 5942534716 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

A delivery agent visits an office once every 5 months. In 2022 he visited the office on March 15. In which year will he next visit the office in March?

Options :

1. ✖ 2025

2. ✖ 2026

3. ✔ 2027

4. ✖ 2028

Question Number : 7 Question Id : 5942534717 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The number of solutions mod 7 of the equation

$X^5 + X^4 + X^3 + X^2 + X \equiv -1 (\text{mod } 7)$ is

Options :

1. ✖ 0

2. ✖ 1

3. ✔ 5

4. ✖ 7

Question Number : 8 Question Id : 5942534718 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement is **NOT** true in a cyclic group G of order n?

Options :

1. ✖ For every divisor m of n, there exists a subgroup of G of order m

2. ✔ The number of subgroups of G is $\phi(n)$

3. ✖ Every subgroup of G is cyclic

4. ✖ Every quotient group of G is cyclic

Question Number : 9 Question Id : 5942534719 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the statements.

S_1 : If H and K are subgroups of a group G then $H \cup K$ is a subgroup if and only if either $H \subseteq K$ or $K \subseteq H$.

S_2 : If H and K are subgroups of a group G the $H \cup K$ is a subgroup if and only if $H \cup K = G$.

Then

Options :

1. ✖ both S_1 and S_2 are false

2. ✔ S_1 is true but S_2 is false

3. ✖ S_1 is false but S_2 is true

4. ✖ both S_1 and S_2 are true

Question Number : 10 Question Id : 5942534720 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The maximum possible order of an element in the permutation group S_{12} is

Options :

1. ✖ 12!

2. ✔ 60

3. ✖ 42

4. ✖ 35

Question Number : 11 Question Id : 5942534721 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following number cannot be the order of any element in the permutation group S_8?

Options :

1. ✔ 9

2. ✖ 15

3. ✖ 6

4. ✖ 7

Question Number : 12 Question Id : 5942534722 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The number of possible homomorphisms from the dihedral group D_{10} to a cyclic group of order 5 is

Options :

1. ✖ 5

2. ✖ 3

3. ✖ 2

4. ✔ 1

Question Number : 13 Question Id : 5942534723 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If C_n is a cyclic group of order n, then the number of proper subgroups of $C_2 \times C_2 \times C_2$ is

Options :

1. ✖ 8

2. ✖ 11

3. ✖ 12

4. ✔ 15

Question Number : 14 Question Id : 5942534724 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the following statements.

S_1 : In a group of order 105 any subgroup of order 35 (if it exists) must be normal.

S_2 : A group of order 27 cannot be simple.

Then

Options :

1. ✖ both S_1 and S_2 are false

2. ✖ S_1 is true but S_2 is false

3. ✖ S_1 is false but S_2 is true

4. ✔ both S_1 and S_2 are true

Question Number : 15 Question Id : 5942534725 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The number of elements of order 15 in a group of order 30 is

Options :

1. ✖ 6

2. ✖ 7

3. ✔ 8

4.

✖ 9

Question Number : 16 Question Id : 5942534726 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The number of conjugacy classes in the permutation group S_6 is

Options :

1. ✖ 6

2. ✖ 8

3. ✖ 10

4. ✔ 11

Question Number : 17 Question Id : 5942534727 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

An example of an infinite group in which every element has finite order

Options :

1. ✖ is $\mathbb{R}/\mathbb{Z}$

2. ✖ is the group of continuous real valued functions on [0,1] under pointwise addition

3. ✔ is $\mathbb{Q}/\mathbb{Z}$

4. ✖ does not exist

Question Number : 18 Question Id : 5942534728 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The number of units in the subring $\mathbb{Z} + \mathbb{Z}\omega = \{a + b\omega \mid a, b \in \mathbb{Z}\}$ $(\omega = e^{\frac{2\pi i}{3}})$ of the complex numbers is

Options :

1. ✔ 6

2. ✖ 4

3. ✖ 3

4. ✖ 2

Question Number : 19 Question Id : 5942534729 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the following statements.

S_1 : Every Euclidean domain is a principal ideal domain.

S_2 : $3+4i$ is a prime element in the subring $\mathbb{Z}+\mathbb{Z}i$ of complex numbers.

Then

Options :

1. ✖ both S_1 and S_2 are false

2. ✔ S_1 is true but S_2 is false

3. ✖ S_1 is false but S_2 is true

4. ✖ both S_1 and S_2 are true

Question Number : 20 Question Id : 5942534730 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following polynomial is irreducible?

Options :

1. ✖ $X^2+\overline{2}$ in $\mathbb{Z}_3[X]$

2. ✔ $X^2+X+\overline{1}$ in $\mathbb{Z}_2[X]$

3. ✖ X^5+1 in $\mathbb{Z}[X]$

4. ✖ $X^2-\overline{3}X+\overline{4}$ in $\mathbb{Z}_7[X]$

Question Number : 21 Question Id : 5942534731 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The number of ideals in the ring $\mathbb{Z}_{99}$ of integers modulo 99 is

Options :

1. ✖ 5

2. ✖ 9

3. ✖ 11

4. ✔ 6

Question Number : 22 Question Id : 5942534732 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following ring has no proper two-sided non-zero ideal?

Options :

1. ✖ $M_2(\mathbb{Z})$

2. ✖ $\mathbb{Z}_{60}$

3. ✖ $M_3(\mathbb{Z})$

4. ✔ $M_2(\mathbb{R})$

Question Number : 23 Question Id : 5942534733 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

In the ring of polynomials $\mathbb{R}[X]$, the ideal generated by $2X^2 - 6X + 4$ and $3X^2 - 15X + 18$ is equal to

Options :

1. ✔ the ideal generated by $X - 2$

2. ✖ the ideal generated by $X - 1$

3. ✖ the ideal generated by $X - 3$

4. ✖ $\mathbb{R}[X]$

Question Number : 24 Question Id : 5942534734 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let R be the ring of continuous functions on the interval [0,1] with pointwise addition and multiplication. Then which of the following statement is true in R?

Options :

1. ✖ R is an integral domain

2. ✖ The ideal generated by $X-1$ is maximal

3. ✖ The ideal generated by X^2+2 is maximal

4. ✔ $\{f \in R \mid f(1)=0\}$ is a maximal ideal

Question Number : 25 Question Id : 5942534735 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The degree of the extension $\mathbb{Q}(e^{\frac{2\pi i}{8}})$ over $\mathbb{Q}(\sqrt{2})$ is

Options :

1. ✖ 1

2. ✔ 2

3. ✖ 4

4. ✖ 8

Question Number : 26 Question Id : 5942534736 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement is **FALSE**?

Options :

1. ✖ A field of 49 elements is a separable extension of its prime subfield of 7 elements

2. ✖ $\mathbb{Q}(\sqrt[3]{2}, e^{\frac{2\pi i}{3}})$ is a normal extension of $\mathbb{Q}$

3. ✖ A quadratic extension of $\mathbb{Q}$ is always a Galois extension of $\mathbb{Q}$

4. ✔ $\mathbb{Q}(\sqrt[4]{2})$ is a Galois extension of $\mathbb{Q}$

Question Number : 27 Question Id : 5942534737 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The Galois group of $\mathbb{Q}(\sqrt[3]{53}, e^{\frac{2\pi i}{3}})$ over $\mathbb{Q}$ is

Options :

1. ✖ a cyclic group C_3 of order 3

2. ✖ a cyclic group of order 53

3. ✖ $C_3 \times C_3$

4. ✔ S_3

Question Number : 28 Question Id : 5942534738 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

In $\mathbb{R}^4$, which of the following vectors is orthogonal to the vectors $(2,1,1,2)$ and $(1,2,2,1)$ with respect to the usual inner product?

Options :

1. ✖ $(1,-1,2,-2)$

2. ✖ $(2,-2,1,-1)$

3. ✖ $(2,1,-1,2)$

4. ✔ $(1,-2,2,-1)$

Question Number : 29 Question Id : 5942534739 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The index and signature of the quadratic form

$X_1^2 - X_2^2 - 2X_3^2 - 5X_4^2 + 2X_5^2$

are respectively

Options :

1. ✔ 2 and -1

2. ✖ 4 and 0

3. ✖ 1 and 4

4. ✖ 4 and 1

Question Number : 30 Question Id : 5942534740 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The rank and nullity of the matrix $\begin{bmatrix} 0 & 0 & 6 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ are respectively

Options :

1. ✖ 2 and 2

2. ✔ 4 and 0

3. ✖ 1 and 3

4. ✖ 3 and 1

Question Number : 31 Question Id : 5942534741 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

In the real vector space of real polynomials of degree less than 5, the dimension of the linear span of $1+X, 1+2X^2, 1+3X^3, 1+4X^4$ is

Options :

1. ✖ 5

2. ✔ 4

3. ✖ 3

4. ✖ 2

Question Number : 32 Question Id : 5942534742 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the following statements.

S_1 : If A is an $n \times n$ real matrix such that $AA^T = I_n$, then the row vectors of A form an orthonormal basis of $\mathbb{R}^n$.

S_2 : If A is an $n \times n$ real matrix such that $AA^T = I_n$, then the determinant of A must be 1.

Then

Options :

1. ✖ both S_1 and S_2 are false

2. ✔ S_1 is true but S_2 is false

3. ✖ S_1 is false but S_2 is true

4. ✖ both S_1 and S_2 are true

Question Number : 33 Question Id : 5942534743 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the real vector space of polynomials over the reals of degree less than 4 with the inner product $< f, g >= \int_0^1 f(x)g(x)\,dx$. Then a pair of orthonormal vectors is

Options :

1. ✔ $1, \sqrt{3}(2X-1)$

2. ✖ $1, \sqrt{3}(2X+1)$

3. ✖ $1, \sqrt{3}(X-2)$

4. ✖ $1, \sqrt{3}(X+2)$

Question Number : 34 Question Id : 5942534744 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If v_1, v_2, v_3 is a basis of a vector space and $T : V \to V$ is a linear transformation such that $T(v_1) = v_2, T(v_2) = v_1, T(v_3) = v_3 - v_1$, then the matrix of T^{20} with respect to the ordered basis $\{v_1, v_2, v_3\}$ is

Options :

1. ✔ $\begin{bmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{bmatrix}$

2. ✖ $\begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 20 \\ 0 & 0 & 1 \end{bmatrix}$

3. ✖ $\begin{bmatrix} 1 & 0 & 0 \\ -10 & 1 & 0 \\ -10 & 1 & 1 \end{bmatrix}$

4. ✖ $\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$

Question Number : 35 Question Id : 5942534745 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, then the polynomial of least degree satisfied by A is

Options :

1. ✖ $(X-1)^4$

2. ✖ $(X-1)^3$

3. ✔ $(X-1)^2$

4. ✖ X^2-1

Question Number : 36 Question Id : 5942534746 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The rank of the real matrix $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 5 \end{bmatrix}$ is

Options :

1. ✖ 1

2. ✔ 2

3. ✖ 3

4. ✖ 4

Question Number : 37 Question Id : 5942534747 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement is FALSE?

Options :

1. ✖ If two matrices represent the same linear transformations with respect to two different bases then the matrices are conjugate

2. ✖ Conjugate matrices have the same characteristic polynomial

3. ✔ The matrices $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ are conjugate

4. ✖ The matrices $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are conjugate

Question Number : 38 Question Id : 5942534748 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation given by
$T((a, b, c)) = (3a + 2b - c, a + 3b + 2c, -2a + b + 3c)$, $(a, b, c) \in \mathbb{R}^3$,
then the rank and nullity of T are respectively

Options :

1. ✖ 2 and 2

2. ✔ 2 and 1

3. ✖ 1 and 2

4. ✖ 3 and 0

Question Number : 39 Question Id : 5942534749 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $A = \begin{bmatrix} \overline{1} & \overline{2} \\ \overline{3} & \overline{4} \end{bmatrix}$ be a matrix in $M_2(\mathbb{Z}_5)$ where $\mathbb{Z}_5$ is the field of 5 elements. Then the additive and multiplicative orders of A are respectively

Options :

1. ✖ 4 and 5

2. ✔ 5 and 8

3. ✖ 8 and 5

4. ✖ 5 and 4

Question Number : 40 Question Id : 5942534750 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The quadratic form $X_1^2 + X_1X_2 - X_3^2$ is

Options :

1. ✖ negative definite

2. ✖ negative semi-definite

3. ✖ positive semi-definite

4. ✔ indefinite

Question Number : 41 Question Id : 5942534751 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement is **FALSE**?

Options :

1. ✖ The rank of an invertible $n \times n$ matrix must be n

2. ✖ The column vectors of an invertible square matrix must be linearly independent

3. ✔ If an $n \times n$ matrix with coefficients in $\mathbb{Z}$ is invertible with inverse also with integer coefficients then its determinant must be 1

4. ✖ The row vectors of an invertible matrix must be linearly independent

Question Number : 42 Question Id : 5942534752 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let M_3 denote the vector space of all 3×3 matrices with complex entries over the field $\mathbb{R}$. If $V = \{A \in M_3 | \ A \text{ is a Hermitian matrix}\}$, then the dimension of $\frac{M_3}{V}$ over the field $\mathbb{R}$ is

Options :

1. ✖ 2

2. ✖ 3

3. ✔ 9

4. ✖ 18

Question Number : 43 Question Id : 5942534753 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If V^* denotes the dual of the vector space $\mathbb{R}^n$ over $\mathbb{R}$, then the dimension of $\{f \in V^* | \ f(1, 0, 0, \ldots, 0) = 0\}$ is

Options :

1. ✖ 1

2. ✔ $n-1$

3. ✖ n

4. ✖ $n+1$

Question Number : 44 Question Id : 5942534754 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $(V, <, >)$ be a finite dimensional inner product space. Consider the statements

S_1 : There exists a basis of V consisting of mutually orthogonal vectors.

S_2 : If $B = \{x_1, x_2, \ldots, x_n\} \subset V, x_i \perp x_j$ for $1 \leq i, j \leq n, i \neq j$, then B is linearly independent

Then

Options :

1. ✖ S_1 is true, S_2 is false

2. ✖ S_2 is true, S_1 is false

3. ✔ S_1 is true, S_2 is true

4. ✖ S_1 is false, S_2 is false

Question Number : 45 Question Id : 5942534755 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $V = C[-1, 1]$, the set of real-valued continuous functions on $[-1, 1]$. Consider V as a vector space over the field $\mathbb{R}$. For $w \in V$, define $< f, g >_w = \int_{-1}^{1} w(x) f(x) g(x) dx, \ \forall f, g \in V$. Which of the following w makes $<, >_w$ an inner product?

Options :

1. ✖ $w(x) = x^3, \forall x \in [-1, 1]$

2. ✖ $w(x) = \sin x, \forall x \in [-1, 1]$

3. ✔ $w(x) = x^2, \forall x \in [-1, 1]$

4. ✖ $w(x) = \sin \pi x, \forall x \in [-1, 1]$

Question Number : 46 Question Id : 5942534756 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement is true?

Options :

1. ✖ The Cantor set is dense in $(0, 1)$

2. ✖ The Cantor set is an open set

3. ✔ The Cantor set is uncountable

4. ✖ The Cantor set is not a perfect set

Question Number : 47 Question Id : 5942534757 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If (a_n) and (b_n) are two bounded sequences and $b_n \neq 0$, $n \in \mathbb{N}$, then which of the following statement is always true?

Options :

1. ✖ Both (a_n) and (b_n) converge

2. ✔ $(a_n + b_n)$ has a convergent subsequence

3. ✖ $(a_n - b_n)$ is convergent

4. ✖ $(\frac{a_n}{b_n})$ is unbounded

Question Number : 48 Question Id : 5942534758 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement is true?

Options :

1. ✖ If a series $\sum_{n=1}^{\infty} x_n$ is convergent, then $\sum_{k=1}^{\infty} x_{n_k}$ converges for every subsequence (x_{n_k}) of (x_n)

2. ✖ At the end points of the interval of convergence of a power series, the series is not convergent

3. ✖ The Cauchy product of two convergent series is convergent

4. ✔ The Cauchy product of two absolutely convergent series is absolutely convergent

Question Number : 49 Question Id : 5942534759 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement need **NOT** be true?

Options :

1. ✖ Composition of two monotonic functions is monotonic

2. ✖ Composition of two uniformly continuous functions is uniformly continuous

3. ✖ Composition of two differentiable functions is differentiable

4. ✔ Composition of two discontinuous functions is discontinuous

Question Number : 50 Question Id : 5942534760 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0, \\ 0, & x = 0, \end{cases}$ then the derivative of f at $x = 0$

Options :

1. ✖ does not exist

2. ✖ exists and is equal to 1

3. ✔ exists and is equal to 0

4. ✖ exists and is equal to 2

Question Number : 51 Question Id : 5942534761 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

$$\left\{x \in \mathbb{R} \middle| \sum_{n=1}^{\infty} \frac{x^n}{n} \text{ converges}\right\} =$$

Options :

1. ✖ $[0, 1]$

2. ✔ $[-1, 1)$

3. ✖ $\{0\}$

4. ✖ $\{1\}$

Question Number : 52 Question Id : 5942534762 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the following statements.

S_1 : If $f : \mathbb{R} \to \mathbb{R}$ is analytic, then f is infinitely differentiable.

S_2: If $f : \mathbb{R} \to \mathbb{R}$ is infinitely differentiable, then f is analytic.

Then

Options :

1. ✔ S_1 is true but S_2 is false

2. ✖ S_1 is false but S_2 is true

3. ✖ both S_1 and S_2 are true

4. ✖ both S_1 and S_2 are false

Question Number : 53 Question Id : 5942534763 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $f, f_n : [0, 1] \to \mathbb{R}$, $n \in \mathbb{N}$, are such that $f_n \to f$ uniformly on $[0, 1]$, then which of the following need **NOT** be true?

Options :

1. ✖ If every f_n is continuous, then f is continuous

2. ✔ If every f_n is differentiable, then f is differentiable

3. ✖ If every f_n is bounded and monotonic, then $\int_0^1 f_n(x)dx \to \int_0^1 f(x)dx$

4. ✖ If every f_n is monotonically increasing, then f is monotonically increasing

Question Number : 54 Question Id : 5942534764 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $f : [0, 1] \to \mathbb{R}$ be a bounded function, $\mathcal{L}(P, f), \mathcal{U}(P, f)$ denote the lower Darboux sum, upper Darboux sum of f with respect to partition P of $[0, 1]$, respectively. If P^* is a refinement of the partition P then which of the following is always true?

Options :

1. ✖ $\mathcal{L}(P^*, f) \geq \mathcal{U}(P^*, f)$

2. ✖ $\mathcal{L}(P^*, f) < \mathcal{L}(P, f)$

3. ✔ $\mathcal{U}(P, f) \geq \mathcal{L}(P^*, f)$

4. ✖ $\mathcal{L}(P^*, f) \geq \mathcal{U}(P, f)$

Question Number : 55 Question Id : 5942534765 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Define $f : [0,1] \to \mathbb{R}$ as

$$f(x) = \begin{cases} 1, & \text{if } x = 0, \\ \frac{1}{n}, & \text{if } x = \frac{m}{n},\ \gcd(m,n) = 1,\ m \in \mathbb{Z},\ n \in \mathbb{N}, \\ 0, & \text{otherwise} \end{cases}$$

then

Options :

1. ✖ f is not Riemann integrable

2. ✔ f is continuous only at irrational numbers in [0,1]

3. ✖ f is discontinuous at every number in [0,1]

4. ✖ f is a function of bounded variation

Question Number : 56 Question Id : 5942534766 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If C is a square with vertices at $2 \pm 2i, -2 \pm 2i$, then $\oint_C \frac{dz}{z-1} =$

Options :

1. ✔ $2\pi i$

2. ✖ 1

3. ✖ 16

4. ✖ $16i$

Question Number : 57 Question Id : 5942534767 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $f(x+iy) = x^2 - y^2, x, y \in \mathbb{R}$, then

Options :

1. ✖ f satisfies the Cauchy–Riemann equations in $\mathbb{C}$

2. ✖ f is entire

3. ✖ f is differentiable in $\mathbb{C} \setminus \{0\}$

4. ✔ f is differentiable only at $z = 0$

Question Number : 58 Question Id : 5942534768 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $f(x+iy) = x^2-y^2+2i\lvert xy\rvert, S = \{z \in \mathbb{C} \mid f \text{ is analytic in a neighbourhood of } z\}$, then $S =$

Options :

1.

✔ $\{x+iy|\ xy>0\}$

2. ✖ $\{x+iy|\ xy<0\}$

3. ✖ $\{x+iy|\ xy\neq 0\}$

4. ✖ $\{x+iy|\ xy=0\}$

Question Number : 59 Question Id : 5942534769 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the following statements:

S_1 : There exists an entire function f such that $f(\mathbb{C})=\{1,2\}$.

S_2 : There exists an entire function f such that $f(\mathbb{C})=\{x+iy|\ xy=0\}$.

Then

Options :

1. ✖ S_1 is true, S_2 is false

2. ✖ S_2 is true, S_1 is false

3. ✖ S_1 is true, S_2 is true

4. ✔ S_1 is false, S_2 is false

Question Number : 60 Question Id : 5942534770 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

$$\oint_{|z|=3} \frac{e^{2z}}{(z+1)^4} dz =$$

Options :

1. ✖ $\frac{8\pi i e^2}{3}$

2. ✔ $\frac{8\pi e^{-2}}{3}$

3. ✖ $8\pi i e^2$

4. ✖ $8\pi i e^{-2}$

Question Number : 61 Question Id : 5942534771 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The region of convergence of $\sum_{n=1}^{\infty} \frac{(z+2)^{n-1}}{(n+1)^3} \cdot \frac{1}{4^n}$ is

Options :

1. ✖ $\{z/\ |z| < 1\}$

2. ✔ $\{z/\ |z+2|<4\}$

3. ✖ $\{z/\ |z-2|<1/4\}$

4. ✖ $\mathbb{C}$

Question Number : 62 Question Id : 5942534772 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The Laurent series of $\frac{z}{(z-1)(2-z)}$ in $|z|>2$ is

Options :

1. ✖ $-\frac{1}{z}-\frac{3}{z^2}+\frac{7}{z^3}-\cdots$

2. ✖ $-\frac{1}{z}+\frac{3}{z^2}+\frac{7}{z^3}+\cdots$

3. ✔ $-\frac{1}{z}-\frac{3}{z^2}-\frac{7}{z^3}-\cdots$

4. ✖ $-\frac{1}{z}+\frac{3}{z^2}-\frac{7}{z^3}+\cdots$

Question Number : 63 Question Id : 5942534773 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the statements:

S_1 : The function $f : \mathbb{C} \to \mathbb{C}, f(z) = \sin z$ is bounded.

S_2 : If f is entire and bounded on two straight lines in $\mathbb{C}$, then f is a constant function.

Then

Options :

1. ✖ S_1 is true, S_2 is false

2. ✖ S_2 is true, S_1 is false

3. ✖ S_1 is true, S_2 is true

4. ✔ S_1 is false, S_2 is false

Question Number : 64 Question Id : 5942534774 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $D_1 = \{z \in \mathbb{C} | \ |z| < 1\}$. Consider the statements:

S_1 : There exists an analytic function $f : D_1 \to \mathbb{C}$ such that the set of all zeros of f is $\{\frac{1}{n} | \ n \in \mathbb{N}\} \cup \{0\}$

S_2 : There exists an analytic function $f : D_1 \to \mathbb{C}$ such that the set of all zeros of f is an infinite subset of $\{z | \ \frac{1}{3} < |z| < \frac{1}{2}\}$.

Then

Options :

1. ✖ S_1 is true, S_2 is false

2. ✖ S_2 is true, S_1 is false

3. ✖ S_1 is true, S_2 is true

4. ✔ S_1 is false, S_2 is false

Question Number : 65 Question Id : 5942534775 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following space is complete?

Options :

1. ✖ $(C[0,1], \| \ \|_1)$, where $C[0,1] = \{f : [0,1] \to \mathbb{R} |\ f \text{ is continuous}\}$,

$$\|f\|_1 = \int_0^1 |f(x)|dx,\ \forall f \in C[0,1]$$

2. ✖ $(C_{00}, \| \ \|_\infty)$, where $C_{00} = \{(x_n) |\ x_n \in \mathbb{R}, \forall n \in \mathbb{N}, \{n |\ x_n \neq 0\} \text{ is finite}\}$,
$\|(x_n)\|_\infty = \max_{n\in\mathbb{N}} |x_n|,\ \forall (x_n) \in C_{00}$

3. ✖ $(C_{00}, \| \ \|_2)$, where $\|(x_n)\|_2 = \left(\sum_{n=1}^{\infty} |x_n|^2\right)^{1/2},\ \forall (x_n) \in C_{00}$

4. ✔ $(C_0, \| \ \|_\infty)$, where $C_0 = \{(x_n) |\ x_n \in \mathbb{R}, \forall n \in \mathbb{N},\ (x_n) \to 0\}$,
$\|(x_n)\|_\infty = \sup_{n\in\mathbb{N}} |x_n|,\ \forall (x_n) \in C_0$

Question Number : 66 Question Id : 5942534776 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $C[0,1] = \{f : [0,1] \to \mathbb{R} |\ f \text{ is continuous}\}$,
$C^1[0,1] = \{f \in C[0,1] |\ f' \text{ is continuous on } (0,1)\}$, $\|f\|_\infty = \sup_{x\in[0,1]} |f(x)|$,
$\|f\|_p = (\int_0^1 |f(x)|^p dx)^{1/p}$, $\forall f \in C[0,1]$. Then which of the following is a bounded linear operator?

Options :

1. ✔ $T : (C[0,1], \|\ \|_\infty) \to (C[0,1], \|\ \|_\infty), (Tf)(x) = \int_0^x f(t)dt$

2. ✖ $T : (C^1[0,1], \|\ \|_\infty) \to (C[0,1], \|\ \|_\infty), Tf = \frac{df}{dx}$

3. ✖ $T : (C[0,1], \|\ \|_1) \to (C[0,1], \|\ \|_\infty), Tf = f$

4. ✖ $T : (C[0,1], \|\ \|_2) \to (C[0,1], \|\ \|_\infty), Tf = f$

Question Number : 67 Question Id : 5942534777 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let H be a Hilbert space and M be a closed subspace of H. Then which of the following is **FALSE**?

Options :

1. ✔ $M \neq M^{\perp\perp}$

2. ✖ If $M^{\perp} = \{0\}$, then M is dense

3. ✖ If M is dense, then $M^{\perp} = \{0\}$

4. ✖ $\forall x \notin M, \exists y \in M$ such that $\|x - y\| = \min_{z \in M} \|x - z\|$

Question Number : 68 Question Id : 5942534778 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $(X, \| \ \|_1), (X, \| \ \|_2)$ be two Banach spaces. Let τ_1, τ_2 be topologies given by $\| \ \|_1, \| \ \|_2$, respectively.

Consider the statements:

S_1 : The identity operator $I : (X, \| \ \|_1) \to (X, \| \ \|_2)$ is a bounded linear operator.

S_2 : If $\|x_n\|_1 \to 0$, then $\|x_n\|_2 \to 0$, for every sequence (x_n) in X.

S_3 : $\tau_1 \subseteq \tau_2$.

Which one or more of the above statements implies $\tau_1 = \tau_2$?

Options :

1. ✖ S_1 only

2. ✖ S_1, S_3 only

3. ✖ S_2 only

4. ✔ $S_1,\ S_2$ and S_3

Question Number : 69 Question Id : 5942534779 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the following statements:

S_1 : If X is a Banach space , Y is a normed space, $T : X \to Y$ is a bounded linear operator, then T is a closed linear operator.

S_2 : If X is a normed space, Y is a Banach space, $T : X \to Y$ is a closed linear operator, then T is a bounded operator.

Then

Options :

1. ✔ S_1 is true and S_2 is false

2. ✖ S_1 is false and S_2 is true

3. ✖ S_1 and S_2 are true

4. ✖ S_1 and S_2 are false

Question Number : 70 Question Id : 5942534780 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let X, Y be Banach spaces. Consider the statements:

S_1 : Let $T_n : X \to Y$ be a bounded linear operator for each $n \in \mathbb{N}$. The sequence of operator norms $(\|T_n\|)$ is bounded if and only if $\forall x \in X$ the sequence of norms $(\|T_n x\|)$ is bounded in Y.

S_2 : If $(f(x_n))$ is bounded, $\forall f \in X'$ (dual of X), then (x_n) is bounded.

Then

Options :

1. ✖ S_1 is true and S_2 is false

2. ✖ S_1 is false and S_2 is true

3. ✔ S_1 and S_2 are true

4. ✖ S_1 and S_2 are false

Question Number : 71 Question Id : 5942534781 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following statement does **NOT** hold true in an arbitrary metric space (X, d)?

Options :

1. ✖ Every Cauchy sequence in X is bounded

2. ✖ (X, d) is complete if and only if every Cauchy sequence in X has a convergent subsequence

3. ✖ If $(x_n), (y_n)$ are Cauchy sequences in X, then $d(x_n, y_n)$ is a convergent sequence

4. ✔ Every bounded sequence has a subsequence which is Cauchy

Question Number : 72 Question Id : 5942534782 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let (X, d) be a metric space, $A \subseteq X, A \neq \emptyset$. Define

$$d_A(x) = \inf\{d(x, y) |\ y \in A\}.$$

Then which of following statement(s) is(are) true?

$S_1 : d_A$ is uniformly continuous.

$S_2 :$ If $d_A(x) = 0, \forall x \in X$ then $A = X$.

Options :

1. ✔ Only S_1

2. ✖ Only S_2

3. ✖ Both S_1 and S_2

4. ✖ Neither S_1 nor S_2

Question Number : 73 Question Id : 5942534783 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let (X, d) be a compact metric space. Which of the following is NOT true?

Options :

1. ✖ X has a countable dense subset

2. ✖ Every sequence in X has a convergent subsequence

3. ✖ Every infinite subset of X has a limit point

4. ✔ If $f : X \to X$ is continuous, then there exists a fixed point of f

Question Number : 74 Question Id : 5942534784 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following metric space (with Euclidean metric) is connected?

Options :

1. ✖ $\mathbb{Q} \times \mathbb{R}$

2. ✖ $\mathbb{Q} \times (\mathbb{R} \setminus \mathbb{Q})$

3. ✔ $\{(x, n) |\ x \in \mathbb{R}, n \in \mathbb{Z}\} \cup \{(0, y) |\ y \in \mathbb{R}\}$

4. ✖ $\{(x, y) |\ xy = 1, x > 0\} \cup \{(0, y) |\ y \in \mathbb{R}\}$

Question Number : 75 Question Id : 5942534785 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider $\mathcal{B}_1 = \{(a, \infty) |\ a \in \mathbb{R}\}$, $\mathcal{B}_2 = \{(-\infty, a) |\ a \in \mathbb{R}\}$, $\mathcal{B}_3 = \{[a, b] |\ a, b \in \mathbb{R}\}$, and $\mathcal{B}_4 = \{(a, \infty) \cup (-\infty, b) |\ a, b \in \mathbb{R}\}$. Then which of the following is a basis for some topology on $\mathbb{R}$?

Options :

1. ✖ $\mathcal{B}_1$

2. ✖ $\mathcal{B}_1, \mathcal{B}_2$

3. ✖ $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3$

4. ✔ $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4$

Question Number : 76 Question Id : 5942534786 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider $(\mathbb{R}, \tau)$, where τ is the cofinite topology. Then

Options :

1. ✖ every non-empty subset of $\mathbb{R}$ is a dense set

2. ✔ every non-empty subset of $\mathbb{R}$ is a compact set

3. ✖ every basis of $(\mathbb{R}, \tau)$ has only countably many elements

4. ✖ $(\mathbb{R}, \tau)$ is metrizable

Question Number : 77 Question Id : 5942534787 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Consider the following statements:

S_1 : If τ_1, τ_2, τ_3 are distinct topologies on $X \neq \emptyset$, then $\tau_1 \cap \tau_2 \cap \tau_3$ is a topology on X.

S_2 : If τ_1, τ_2, τ_3 are distinct topologies on $X \neq \emptyset$, then $\tau_1 \cup \tau_2 \cup \tau_3$ is a topology on X.

Then

Options :

1. ✖ S_1 is false but S_2 is true

2. ✔ S_1 is true but S_2 is false

3. ✖ both S_1 and S_2 are true

4. ✖ both S_1 and S_2 are false

Question Number : 78 Question Id : 5942534788 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Which of the following defines a metric on $\mathbb{R}$?

Options :

1. ✖ $d(x,y) = |x^2 - y^2|,\ \forall x, y \in \mathbb{R}$

2. ✖ $d(x,y) = 1 + |x^2 - y^2|,\ \forall x, y \in \mathbb{R}$

3. ✖ $d(x,y) = 1 + |x - y|,\ \forall x, y \in \mathbb{R}$

4. ✔ $d(x,y) = \dfrac{|x-y|}{1+|x-y|},\ \forall x, y \in \mathbb{R}$

Question Number : 79 Question Id : 5942534789 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The solution of the differential equation $\dfrac{dx}{dt} + kx^2 = 0$ is

Options :

1. ✖ $\dfrac{1}{x} = kx + c$, where c is an arbitrary constant

2. ✖ $\dfrac{1}{x} = -kx + c$, where c is an arbitrary constant

3. ✔ $\dfrac{1}{x} = kt + c$, where c is an arbitrary constant

4. ✖ $\frac{1}{x} = -kt + c$, where c is an arbitrary constant

Question Number : 80 Question Id : 5942534790 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $a \in \mathbb{R}$, then the solution of $\left(\log \frac{dy}{dx}\right) - a = 0$ is

Options :

1. ✔ $y = xe^a + c$, where c is an arbitrary constant

2. ✖ $x = ye^a + c$, where c is an arbitrary constant

3. ✖ $y = \log x + c$, where c is an arbitrary constant

4. ✖ $x = \log y + c$, where c is an arbitrary constant

Question Number : 81 Question Id : 5942534791 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The degree and the order of the ordinary differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^4}$ are respectively

Options :

1.

✖ 2 and 3

2. ✖ 2 and 4

3. ✖ 1 and 4

4. ✔ 2 and 2

Question Number : 82 Question Id : 5942534792 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $\frac{dy}{dx} + 7x^2y = 0$, $y(0) = \frac{3}{7}$, then $y(1) =$

Options :

1. ✖ $\frac{7}{3}e^{-\frac{7}{3}}$

2. ✖ $\frac{7}{3}e^{-\frac{3}{7}}$

3. ✔ $\frac{3}{7}e^{-\frac{7}{3}}$

4. ✖ $\frac{3}{7}e^{-\frac{3}{7}}$

Question Number : 83 Question Id : 5942534793 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $y = x^2$ is a solution of $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$, then another solution of the same differential equation from the following is

Options :

1. ✖ x^3

2. ✔ $x^2 \log x$

3. ✖ $x \log x$

4. ✖ $x^3 \log x$

Question Number : 84 Question Id : 5942534794 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If the Wronskian solutions of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + Q(x)y = 0$ at $x = 0$ is equal to 1, then the Wronskian is

Options :

1. ✔ e^{4x}

2. ✖ e^{2x}

3. ✖ e^x

4. ✖ e^{3x}

Question Number : 85 Question Id : 5942534795 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If e^x and xe^x are solutions of $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + 4y = 0$, then $P =$

Options :

1. ✖ -1

2. ✔ -2

3. ✖ 1

4. ✖ 2

Question Number : 86 Question Id : 5942534796 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

A particular integral of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^x$ is

Options :

1. ✖ $y = -\left(1 + x + \frac{x^2}{3}\right) e^x$

2. ✖ $y = \left(1 + x + \frac{x^2}{2}\right) e^x$

3. ✔ $y = -\left(1 + x + \frac{x^2}{2}\right) e^x$

4. ✖ $y = -\left(1 + x + x^2\right) e^x$

Question Number : 87 Question Id : 5942534797 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

A particular integral of $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} \sin x$ is

Options :

1. ✖ $-e^{2x} \cos x$

2. ✔ $e^{2x} \cos x$

3. ✖ $-e^{2x} \sin x$

4.

✖ $e^{2x}\sin x$

Question Number : 88 Question Id : 5942534798 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The general solution of the $x(y-z)\frac{\partial z}{\partial x}+y(z-x)\frac{\partial z}{\partial y}=z(x-y)$ is

Options :

1. ✔ $\phi(x+y+z,xyz)=0$, where ϕ is a C^1 function

2. ✖ $\phi(x+2y+z,xz)=0$, where ϕ is a C^1 function

3. ✖ $x+y+z=\phi(xy)=0$, where ϕ is a C^1 function

4. ✖ $xyz=\phi(2x+y+z)$, where ϕ is a C^1 function

Question Number : 89 Question Id : 5942534799 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The partial differential equation $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+4=0$

Options :

1. ✖ has a unique solution

2. ✖ has more than one solution

3. ✔ does not have solution

4. ✖ has only trivial solution

Question Number : 90 Question Id : 5942534800 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$, then the complete integral of $z^2(1+p^2+q^2) = 1$ is

Options :

1. ✔ $(x-a)^2 + (y-b)^2 + z^2 = 1$, where $a, b \in \mathbb{R}$

2. ✖ $(x-a)^2 + (y-b)^2 + z^2 = 4$, where $a, b \in \mathbb{R}$

3. ✖ $(x-a)^2 + (y-b)^2 + z^2 = 6$, where $a, b \in \mathbb{R}$

4. ✖ $(x-a)^2 + (y-b)^2 + z^2 = 8$, where $a, b \in \mathbb{R}$

Question Number : 91 Question Id : 5942534801 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

A general solution of $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ is

Options :

1. ✖ $F(x+t) + F(x-t)$, where F is an arbitrary C^1 function

2. ✖ $F(x^2 - t^2)$, where F is an arbitrary C^1 function

3. ✔ $F(x+t) + G(x-t)$, where F and G are arbitrary C^1 functions

4. ✖ $G(x+t) + G(x^2 - t^2)$, where F and G are arbitrary C^1 functions

Question Number : 92 Question Id : 5942534802 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. Then the number of integral surfaces of $p^2 + q^2 + 2(p-x)(q-y) = 2z$ passing through the x-axis is

Options :

1. ✖ unique

2. ✔ two

3. ✖ three

4. ✖ more than three

Question Number : 93 Question Id : 5942534803 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $\mathcal{D}$ be a bounded, open, connected set in $\mathbb{R}^2$ and $\partial\mathcal{D}$ denote the boundary of $\mathcal{D}$. Let a non-constant function $u : \mathcal{D} \cup \partial\mathcal{D} \to \mathbb{R}$ be continuous, and harmonic in $\mathcal{D}$. Then u attains its

Options :

1. ✖ maximum in $\mathcal{D}$ and minimum on $\partial\mathcal{D}$

2. ✔ maximum and minimum on $\partial\mathcal{D}$

3. ✖ maximum and minimum in $\mathcal{D}$

4. ✖ minimum in $\mathcal{D}$ and maximum on $\partial\mathcal{D}$

Question Number : 94 Question Id : 5942534804 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The solution of $\dfrac{\partial^3 z}{\partial x^3} - 3\dfrac{\partial^3 z}{\partial x^2 \partial y} + 4\dfrac{\partial^3 z}{\partial y^3} = 0$ is

Options :

1. ✖ $f_1(y-x) + f_2(y+2x) + x f_3(y-2x)$, where f_1, f_2, f_3 are arbitrary C^3 functions

2. ✖ $f_1(y+x)+f_2(y-2x)+xf_3(y+2x)$, where f_1, f_2, f_3 are arbitrary C^3 functions

3. ✖ $f_1(y+x)+xf_2(y+x)+xf_3(y+2x)$, where f_1, f_2, f_3 are arbitrary C^3 functions

4. ✔ $f_1(y-x)+f_2(y+2x)+xf_3(y+2x)$, where f_1, f_2, f_3 are arbitrary C^3 functions

Question Number : 95 Question Id : 5942534805 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The partial differential equation $5\frac{\partial^2 z}{\partial x^2}+6\frac{\partial^2 z}{\partial y^2}=xy$ is

Options :

1. ✔ elliptic type

2. ✖ parabolic type

3. ✖ hyperbolic type

4. ✖ mixed type

Question Number : 96 Question Id : 5942534806 Question Type : MCQ Option Shuffling : Yes Is

Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

If $u(x,t)$ is a solution of $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial t^2} = 0$, and $u(x,0) = x^2$, $\frac{\partial u}{\partial t}(x,0) = 0$, $0 \leq x \leq \infty$, then $u(2,3)$ is equal to

Options :

1. ✖ 20

2. ✖ 40

3. ✖ 60

4. ✔ 80

Question Number : 97 Question Id : 5942534807 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

Let $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. Then a solution of $pqz = p^2(xq + p^2) + q^2(yp + q^2)$ is

Options :

1. ✖ $z = ax + by$, where $a, b \in \mathbb{R}$

2. ✔ $z = ax + by + \frac{a^4 + b^4}{ab}$, where $a, b \in \mathbb{R} \setminus \{0\}$

3. ✖ $z = a(x + y) + b$, where $a, b \in \mathbb{R}$

4. ✖ $z = ax + b^2(y + 1)$, where $a > 1$, $b \in \mathbb{R}$

Question Number : 98 Question Id : 5942534808 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The equation of the plane which is tangent to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ and parallel to the plane $2x + 2y = z$ is

Options :

1. ✖ $2x + 2y - z = 10$

2. ✔ $2x + 2y - z = 8$

3. ✖ $2x + 2y - z = 6$

4. ✖ $2x + 2y - z = 4$

Question Number : 99 Question Id : 5942534809 Question Type : MCQ Option Shuffling : Yes Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is parallel to the plane

Options :

1. ✔ $2x+y-2z=0$

2. ✖ $3x+4y+5z=7$

3. ✖ $x+y+z=2$

4. ✖ $2x+3y+4z=0$

Question Number : 100 Question Id : 5942534810 Question Type : MCQ Option Shuffling : Yes

Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A

Minimum Instruction Time : 0

Correct Marks : 1 Wrong Marks : 0.25

The equation of the right circular cone whose vertex is at the origin, the axis along the x-axis and the semi vertical angle α is

Options :

1. ✖ $x^2+y^2=z^2\tan^2\alpha$

2. ✔ $z^2+y^2=x^2\tan^2\alpha$

3. ✖ $x^2+z^2=y^2\tan^2\alpha$

4. ✖ $(z^2+y^2)\tan^2\alpha=x^2$